Reaction Wheel Force/Torque Disturbance Model Update

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The reaction wheel force and torque disturbance model developed by JPL [1] is an adequate model only for reaction wheels maintained at constant speed. For the purpose of simulation and evaluation of NGST integrated system performance, a more general reaction wheel disturbance model is required. In this memo, a simple formulation of the reaction wheel imbalance is described and used in justifying the two proposed extensions to the current model to allow handling of an arbitrary wheel speed profile.

Consider a simplified model of a reaction wheel that has a single lumped mass imbalance m located at r_0 in the wheel frame whose origin is at the wheel axis, as depicted in the Figure 1. This model is used to explain forces measured in the rotor plane, commonly referred to as "static imbalance" [2]. A slightly different representation is used to explain torques measured in the rotor plane, or "dynamic imbalance".
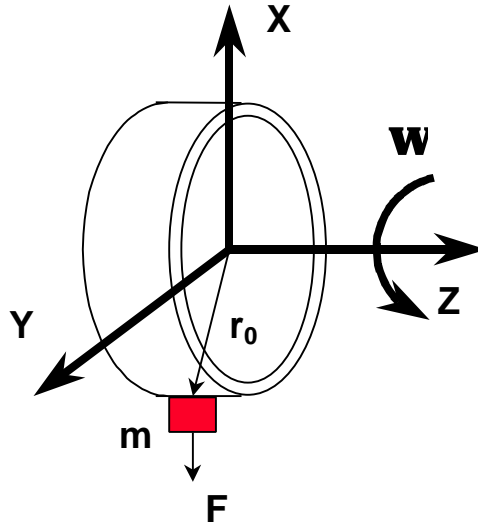

Figure 1 – Simple Imbalance Mass Model

The motion of this lumped mass follows that of the reaction wheel, and can simply be expressed in a fixed frame as:

$$\vec{d}(t) = |r_o|\left(cos(q(t))\hat{i} + sin(q(t))\hat{j}\right)$$

where $q(t)$ represents the rotational angle along the reaction wheel spin axis, assumed here to be the z-axis of the fixed frame.

This expression can be given in matrix notation as:

$$d = R(t)r_o$$

where $R(t)$ is the transformation matrix to the fixed frame from the reaction wheel frame which moves with the reaction wheel rotation, and r_o the column vector. Taking the first and second derivatives of this expression yields:

$$\dot{d} = [\dot{q}]R(t)r_o$$

$$\ddot{d} = \left([\ddot{q}] + [\dot{q}]^2\right)R(t)r_o$$

where the squared bracket denotes the cross product operator matrix, i.e. [a]b=a×b.

The tangential component of the imbalance force is therefore:

$$F_t = m\ddot{d}$$

For a constant reaction wheel speed, i.e. $\ddot{q} = 0$, the above equation when expressed explicitly in vector form reduces to:

$$F_t = m[\dot{q}]^2 R(t)r_o = -|r_o||\dot{q}|^2\left(cos(q(t))\hat{i} + sin(q(t))\hat{j}\right)$$

By following a similar approach, other disturbance components (radial force, in-plane torques) can be derived, and can be shown to have the same functional form as the tangential component of the imbalance force, and differ only by some constant factors.

In the JPL model, the force and torque are given in terms of discrete harmonics of the reaction speed, f_{rwa} (expressed in Hz), and are described by the following expression:

$$m(t) = \sum_{i}^{n} C_i f_{rwa}^2 sin(2ph_i f_{rwa}t + f_i) \quad \text{(equation A)}$$

where $m(t)$ represents some arbitrary component of either disturbance torque or force, and C_i, h_i, and f_i, $(i=1..n)$, are amplitude coefficients, harmonic numbers, and random phases to be extracted from test measurements. In this form, the tangential component of the imbalance force derived above for a lumped mass essentially agrees with the model for the constant speed case. This is true because in this case:

$$q(t) = \int^{t} \dot{q}(t)dt = 2pf_{rwa}t$$

However, when the reaction speed is changing the model must be modified not only to take into account the acceleration term but also to include the more general computation of the rotational angle, i.e. as integration of the reaction wheel speed instead of the product of it and time.

The proposed changes result in the following model of the force and torque disturbance with f_{rwa} now being a time-dependent function:

$$m(t) = \sum_{i}^{n} C_i \left(f_{rwa}^2(t) \, \sin\left(2\boldsymbol{ph}_i \int^{t} f_{rwa}(t') dt' + \boldsymbol{f}_i \right) - \dot{f}_{rwa}(t) \, \cos\left(2\boldsymbol{ph}_i \int^{t} f_{rwa}(t') dt' + \boldsymbol{f}_i \right) \right) \quad \text{(equation B)}$$

Equations A and B were implemented in a simple Simulink™ system, shown in Figure 2, and then simulated. Figure 3 is a plot of a continuous version of a step-down rate profile. It is representative of typical wheel speed profiles encountered at the end of fixed-rate slews, and is used to drive the simulation. Equations (A) and (B) are implemented using a simplified harmonic model (a single harmonic with coefficients $C_0=1$, $h_0=1$, $\phi_0=0$). The resulting normalized forces are shown in Figures 4 and 5, respectively.
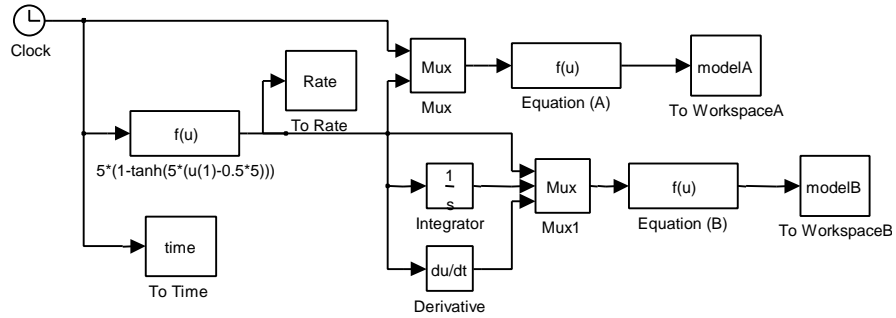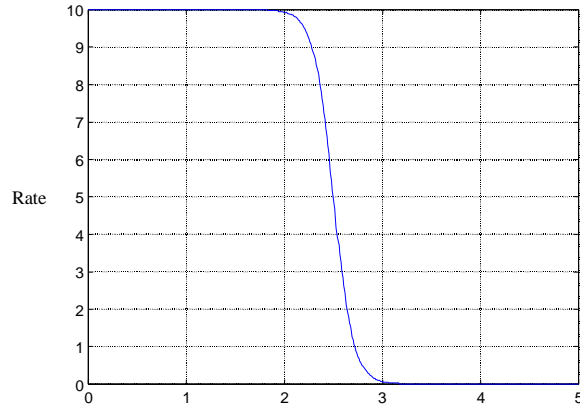

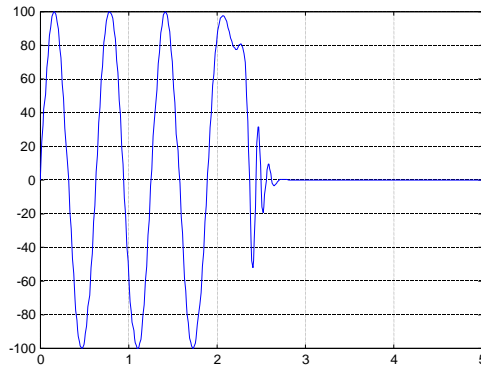Figure 2 – Simulink™ Block Diagram

Figure 3 – Wheel Rate Profile
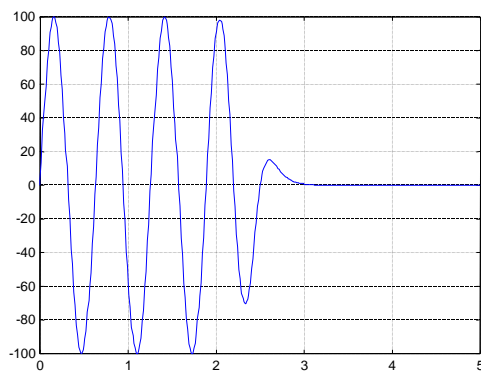
3

Figure 4 – incorrect force modeled using Eqn A

Figure 5 – correct force modeled using Eqn B

References

1. Melody, J.W., "Discrete-Frequency and Broadband Reaction Wheel Disturbance Models," Jet Propulsion Laboratory Interoffice Memorandum 3411-95-200csi, June 1, 1995
2. Bialke, B., "Microvibration Disturbance Sources in Reaction Wheels and Momentum Wheels," Proc. Conference on Spacecraft Structures, Materials & Mechanical Testing, Noordwijk, The Netherlands, March 27-29, 1996